

# A SIMULATION STUDY OF URBAN GROWTH PATTERNS WITH FRACTAL GEOMETRY

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**Abstract.** This paper depicts the use of fractal geometry in urban simulation. Fractal geometry, L-system, the DLA model, and related urban growth theories are first examined. Then an urban simulation system based on fractal geometry and L-system, Fractal\_US, is built on the web for studying urban development patterns in various conditions. The Taipei city is simulated to demonstrate the visualization of urban growth and the result is presented for further discussion.

## 1. Introduction

Understanding urban growth patterns is important for planners to analyze current phenomena and predict the future development. Previous studies about urban growth can be classified into two categories: one provided the prototypes of urban forms, and the other explained their growth processes (Chiu 1998, 1997). Based on these studies, we may understand how cities are developed and affected by the economic, social, and regulatory issues in urban growth processes, but we still cannot visualize what the cities are. For this reason, the goal of this paper is to visualize the urban growth patterns and provide a tool of simulation and prediction for planners.

Geometric regularity is the basis for designers to initiate the formation of urban forms. This paper illustrates a computational approach to constructing the regularity of urban form through fractal geometry. Specifically, the geometric structure underlying urban form is examined. The results of the examination provide a ground for the discussion on the application of fractal geometry to visual simulation, and its implications for future CAAD research.

## 2. Related Theories

This section reviews related theories of form generation including fractal geometry, L-systems, and the diffusion-limited aggregation model; and summarizes related urban studies about urban growth and urban generation patterns (Batty & Longley 1994, Bovill 1996).

### 2.1 FRACTAL GEOMETRY AND L-SYSTEM

#### 2.1.1 Fractal geometry

Fractal geometry, a mechanism for studying objects' forms presented by Mandelbrot in 1967, provides complicated objects possessing of form of self-similarity with explicit descriptions and mathematical models. Self-similarity, one of the most important characteristics of fractal geometry, means complicated geometric objects have similar structures after magnified or reduced at specified scale range called scale-invariant. We then use L-systems to "visualize" urban growth models for its' self-similarity of fractal and growth patterns as plants (Prusinkiewicz and Hanan 1989).

#### 2.1.2 L-systems

L-systems are a mathematical formalism that was proposed by Aristid Lindenmayer in 1968 as a foundation for an axiomatic theory of development. The concept of rewriting is central to the notion of L-systems. The basic idea is to define complex objects by successively replacing parts of a simple initial object using a set of rewriting rulers or productions (Prusinkiewicz and Hanan 1989). As shown in Figure 1, rewriting systems are the central notion of L-systems. We first used AutoLisp to test and visualize form generation. The value of "r" is the number of rewriting or replacement. Ideally, the form should be a perfect geometry, while at  $r = 5$  and  $7$  the form is out of range due to the overlapping problems.

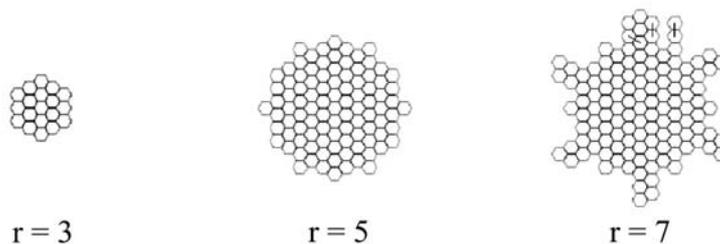


Figure 1. Rewriting systems in generic urban patterns

A different research direction was taken in 1984 by A. R. Smith, who proposed L-systems as a tool for synthesizing realistic images of plants and pointed out the relationship between L-systems and the concept of fractals introduced by Benoit Mandelbrot. The work by Smith inspired our studies of the application of L-systems to computer graphics (Prusinkiewicz and Hanan 1989)

### *2.1.3 Diffusion-Limited Aggregation (DLA)*

The diffusion-limited aggregation model (DLA), an application derived from fractal geometry, was presented by Witten and Sander in 1981. It was cited to the issues of distributions of structure of urban space. DLA refers to the concept of geographic boundaries while fractals can't be specified with L-systems. However, the development of urban space is always influenced by geographic limitation or boundaries. With DLA, the generation will not be continued on the edge of boundaries. Therefore, it's more appropriate to use DLA to illustrate the distribution of structure of urban space (Hsu 1994). Planners can specify the areas or edge boundaries to control the generation.

## 2.2 THE STRUCTURE OF URBAN SPACE

The urban land use theories could be conceptualized into two categories: one is describable (it particularly emphasizes the importance on "what urban form is"), and illustrates the patterns of urban generation. The other is explanatory ("why and how cities grow" is much more important), and focuses on the theories describing the process of urban growth.

### *2.2.1 The urban generation patterns*

The growth of a city is an evolution process, and a city at different stages of development has different urban phenomena. If we study different situations at the series of time, we might find the patterns of urban generation. The models of urban growth can be simulated based on urban studies. While there are numerous theories for researching urban space, but only concentric zones (Burgess 1923), sector hypothesis (Hoyt 1939), and multiple nuclei (Harris and Ullman 1945) are concentrating on the patterns of urban development.

### *2.2.2 The urban growth theories*

The urban ecology is a complex system. A model of urban growth must be able to consider the factors of economics, society, essence and ecology. Generally, the characters of movement, openness, and feedbacks are taken into accounts, while it is hard to predict the change of various activities. Many urban growth theories are explaining why cities grow, how they grow, and how the growth is influenced by various factors as well as the relationship between changes of special structure and growth, and the relationship between urban growth and the country

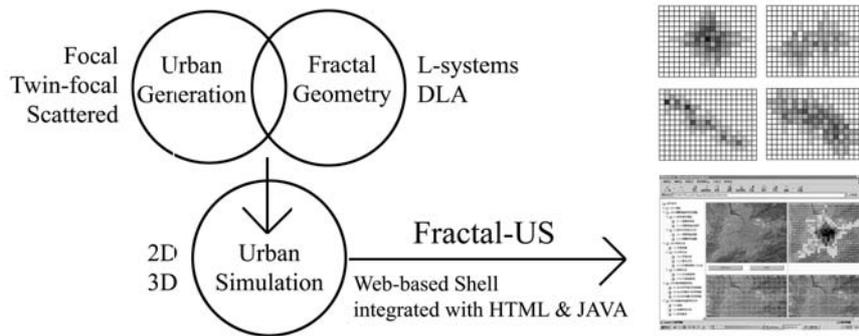
development. However, we are interested in the spatial structure that explains urban growth phenomena, such as the central place theory (Christaller 1933), economic base theory (Hoyt and Andrews), human ecological approach, and communication theory (Meier 1962), while these are not capable to describe the major situations in urban growth.

### 3. Urban Simulation Study

Based on fractal geometry and L-system, we can simulate complicated geometry. At first, we presume urban growth patterns as a focal plan, a twin focal plan, or a scattered plan by means of analyzing the distribution of urban population, and then, use the city of Taipei as an example for simulation. By reviewing the process of Taipei's growth, to find out the regularity and critical factors in the growth process. Then using fractal geometry theory to transform these factors into L-system's operational rules in computers. Because urban growth is generally limited by geographic boundaries, we then use DLA to restrain our simulation of the urban growth (Fotheringham, Batty and Longley 1989). From comparison between the results of simulation and the city, we revise the growth rules to fit the actual case. Through this procedure, we may trace the trend of urban growth, and furthermore predict the future development. Meanwhile, the simulation system is built on the web for educational and demonstration purposes. The details are described below.

#### 3.1 A RESEARCH FRAMEWORK

This study focuses on the application of fractal geometry to urban generation. As shown in Figure 2, a web-based shell is integrated with HTML and JAVA that enable further simulation. Users can specify the focal point, define the growth boundary, or simplify the spatial dimension by a grid system. Various iterations can be generated according to the number assigned.



*Figure 2.* The framework of Fractal-US

### 3.2 THE ANALYSIS OF URBAN GROWTH

Urban growth is a complex process, but all we could do initially is to simplify it. And then we must know what factors affect urban growth, and classify them. Finally, we use all data to simulate it. There are four steps as following.

- (1) Prototypes: A prototype is first built in accordance with three urban patterns such as a focal plan, a twin-focal plan, and a scattered plan, as shown in Figure 3.
- (2) Attraction and Limitation: We can classify all factors that affect urban growth into two categories in terms of their influence on growth: one is helpful to urban growth, and the other is to stop it. And furthermore, both they act upon urban inner in the meantime. The former called "attraction" includes CBD, main transportation, and so on, and the latter called "limitation" includes artificial zoning and actual geographic boundary. L-system in itself is a kind of "attractive" behavior, so we need DLA plays the role of "limitation." Based on the prototype, the development is attracted by directional growth as linear and banding plan, as shown in Figure 4. The development is limited by specific area, as shown in Figure 5.
- (3) Analysis by spatial layers: The factors which affect urban growth are numerous, but in order to discuss individual factor's influence on cities, we have no choice but to divide them. After discussing individual generations, to pile up these factors like cellophanes, and then add all forces by revising their proportion of the whole to adapt physical situations. Finally, the result must be close to the physical situation.
- (4) Subdivision: Simplification is the first step to analyze complex objects. But overly simplification not only loses the true feature, but also diverges from the concept of fractals, and comes back to Euclid geometry. To divide vertically the area into grid units, as shown in Figure 6. Certainly, finer the division is, more close to the complex true, but relatively, the amount of work is getting heavier. So we must determine on the appropriate range of division, and the related factors, like the scale of the object, available related data, and the pixels of the computer.

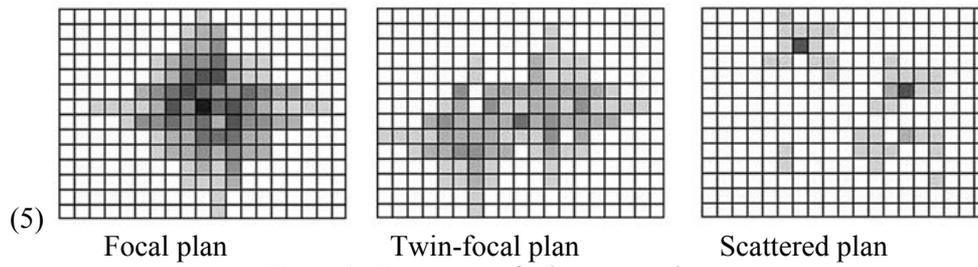


Figure 3. Prototypes of urban generation

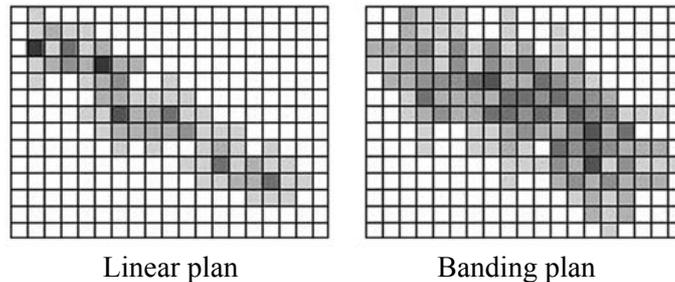


Figure 4. The model of attraction of urban generation

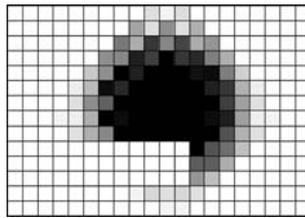


Figure 5. The model of limitation of urban generation

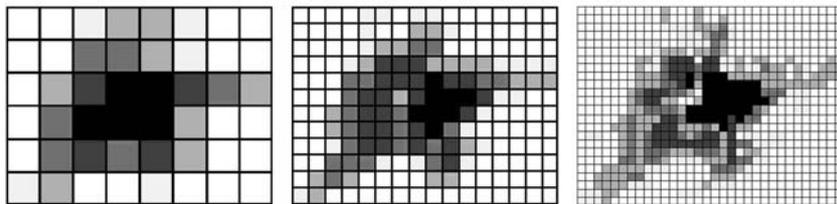


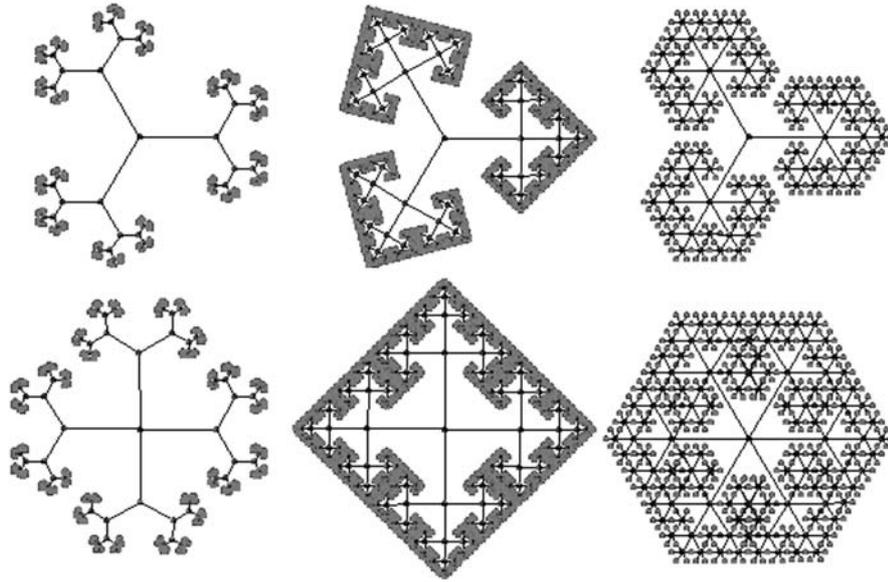
Figure 6. The difference of simulation based various grid size

### 3.3 METHODS FOR REPRESENTATIONS

At the beginning, we use lines to indicate development directions and nodes as focus in simulating the patterns of growth like Central Place Theory (Figure 7), but they don't distinguish the difference of density significantly. Therefore, we draw grid in the plan and express the difference with gray scales (Figure 3-6). The operational tool "AutoLisp" can only generate the recursion of replacement. Because the initial goal is to establish a tool of visual urban simulation that is

operated by anyone, let planners to contact and learn easily this tool is another task of this research. Therefore we use program language “Java” afterward, and put it on web to achieve this proposal (Figure 2).

Cities are three dimensions organisms, so any behavior attempting to explain urban growth by two dimensions plan finally misunderstand the true urban feature. For this reason, this research' latter part will concentrate our attention on establishing 3D models. To 3D, L-systems are still suitable for use for L-systems were used to simulate growth of plants many years ago (Prusinkiewicz and hanan 1989).



*Figure 7.* The simulation of generation used lines as main routes and nodes as places people gather.

#### 4. A Case Study of The Taipei City

Cities are complicated systems, and it is difficult to study all influencing factors such as economics, communications, or geographic boundaries. Therefore, how to select important factors in this research is the major task, while all these factors can be transformed into urban growth rules (or constraints). As defined before, there are two forces acting on models of urban growth: attraction and limitation. So the attraction adopting there includes main transportations, economic

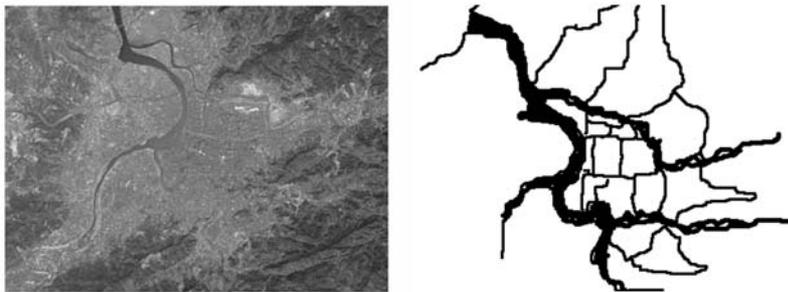
development, development areas, and the regulations of building and zoning, and geographic conditions.

#### 4.1 THE TAIPEI CITY

Based on fractal geometry and L-system, we can simulate complex geometric objects. At first, to divide Taipei's history into two parts in accordance with available data, and the former is the use for experiment on generative rules, and the latter is the use for experiment on fractal geometry's interpretation on simulations of urban growth. And then, we hypothesize patterns of urban growth are focal, twin-focal, or scattered by distribution of urban population.

At the first half of process, based on changes of population distribution, physical plan of development, regulations of laws, geographic conditions, and economic development (Figure 8), we find out rulers of urban generation from these data, simulate by these rulers, and revise the rulers when difference between results of simulation and physical situation exists until getting a suitable model. This procedure is a period of experiment for the most part. After we finish this procedure, we have had ability to find rulers and simulate. The next part is to proof how the fractal geometry's interpretation on the simulation of urban growth is.

In the second period, only based on regulations of laws, geographic condition, and economic development, we don't consider any data about built form, and find out the rulers from these data and simulate it. The similarity between the result and physical form is fractal geometry's interpretation on simulation of urban growth. Actually, there will be lots of situations that might be different because of our methods, just like that may we use built plan or population distribution as urban development? Or the scale of the grid plays an important role on simulation, bigger the scale is, greater the error is getting (Figure 9).



*Figure 8.* The Taipei city Airphoto (Left) and The Taipei city map (Right)

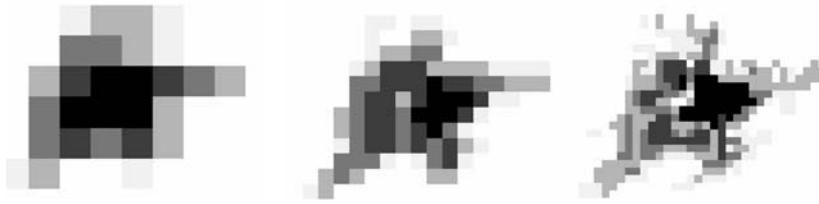


Figure 9. The different scale of simulation in the Taipei County

#### 4.2 THE USE OF DLA

Different limitations have different effects on urban growth. The natural boundaries, like rivers, seashores, or mountains, can slow down growth, but sometimes the behaviors of growth become more active on the edge of geographic boundaries, like ports at the riversides or seashores. DLA defines the attributes of the geographic boundaries, like “slow-down” development near rivers, seas, and mountainsides, and “active” development near ports and bays. While the simulation of growth is close to the lines, the behaviors of growth change according to the attributes of lines. As for artificial limitations, we can define attributes of districts. For example, we predefine the growth control of each district by density or total floor areas, and the growth will be stopped or slow down when the limit is reached.

### 5. Discussion

The study of urban growth patterns with fractal geometry consists of four parts in the process, i.e., literatures/theory reviews, simulation, representation, and interpretation. The above studies constitute a ground for discussion as follows.

- (1) Theory/model: This study demonstrates that urban forms can be self-similar and could be simulated by fractals. The fractal numerical taxonomy is used for clustering geographic instances on the basis of regularity and similarity. The geographic conditions are simplified in simulation for testing the simplest laws of urban formation. The DLA model is proofed to be useful in the study of geographic boundary, while still requires more in-depth studies. Meanwhile, the theory of cellular automata can be compared to the traditional fractal approach to examine the rational of artificial growth (Peitgen, Jurgens, and Saupe 1992).
- (2) Simulation: Computer visual simulation can better illustrate the evolution of urban formation, and consequently understand and discuss the process or impacts in certain areas.
- (3) Representations: The results of simulation in this study are based on 2D representations, and we are working with 3D representation that will help to

achieve the “visual” goal for reproducing the process of urban growth. Urban density can be color-coded or using gray scale to highlight the distribution.

- (4) Interpretation: We apply the concept of layering geographic images to analyze different key elements in geographic conditions such as urban geography, urban zones, traffic systems, etc. Nevertheless, the capability of interpretation or prediction of certain phenomena is still difficult due to only few factors that we are able to simulate or considered in the early simulation. The limitation of representation can be improved by comparison with quantitative models simultaneously.
- (5) Applications: The concept of fractal cities can be applied to the real world (Batty 1994). Meanwhile, the Internet is widely spread and the flow of information on web can be easily distributed among the design and planning society, the simulation system, Fractal-US, can be used for educational and demonstration purposes.

Future directions for this research include (1) the improvement of the analytic capability of Fractal-US, and its extension to an urban planning analytic tool; (2) different scales of urban geography can be studied in depth and incorporated into the system; and (3) cases can be collected for clustering urban patterns.

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